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The use of ARMA models to measure damage potential in seismic records

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ABSTRACT: To assess the damage potential in seismic records, ARMA models are developed with parameters chosen to fit accelerograms of particular earthquake records using maximum likelihood techniques. A random set of accelerograms is generated for each event and used to establish statistically valid structural response spectra. From a sample of earthquakes, the mean and variance of response spectral ordinates are obtained for damage predictors including peak linear displacement, ductility demand and hysteretic energy demand and compared to spectra based on single records.

1. INTRODUCTION

A fundamental element in conventional design for earthquakes is response spectra for single degree of freedom systems (4,14). To establish these spectra, acceleration records from particular earthquakes are used as input to linear and nonlinear models and response measures such as maximum displacement are calculated. For nonlinear structures, the ratio of maximum displacement to yield displacement or ductility factor is used as a design parameter. For any single record, the irregular response spectra obtained are normally smoothed into tripartite linear approximations (15).

When records from different earthquakes are used, the ordinates of design spectra show a good deal of variability. To determine the mean and variance of spectral ordinates, records from different earthquakes are normalized with respect to peak motion characteristics such as peak acceleration or velocity (13). From seismological and geological data, hazard maps giving peak acceleration and velocities with a specified probability of being exceeded in a specified period of time are used to scale the smoothed spectra for design purposes (2).

This approach has a number of serious limitations. Firstly, it is well known that peak ground motions are weak predictors of damage for linear and nonlinear systems (12). Short bursts of very strong acceleration may have little impact on response. Realistic damage prediction requires consideration of the duration and frequency content of seismic records. As well, records which display more than one interval of significant activity contain more damage potential than records with a single segment of strong activity. For these reasons., the subjective concept of effective peak acceleration and velocity have been developed (1).

Secondly, the combination of a set of records from different events is statistically questionable. Records from different events are samples drawn from different populations of source excitation and so do not form a statistically homogeneous basis for analysis.

Thirdly, the use of peak ground motions from an accelerogram as parameters for normalizing response spectra is doubtful. Peak values from one sample of a random process are themselves random variables and, normalization of a random record by a random point on that record introduces an unknown and undesirable measure of uncertainty.

Finally, since peak ground motions are weak predictors of response and are themselves random values of the fundamental processes involved, it will be difficult to establish a valid correlation between damage potential and basic seismological properties such as the seismic moment of earthquake events.

The purpose of the study summarized in this paper is to explore the use of ARMA models as an alternative basis for structural response prediction. Individual records for an earthquake are treated as one realization of an underlying nonstationary random process. The parameters of this stochastic process are estimated from the measured record using maximum likelihood techniques. With these parameters, a sample of accelerograms corresponding to the real event (e.g. El Centro) is generated and used to develop a sample of response spectra for the event. The mean and variance of ordinates to the response spectra can then be estimated with appropriate confidence intervals.

On the basis of such an approach, the relationship between the characteristics of the underlying earthquake process and damage potential can be studied. Eventually, it may be possible to relate the properties of the basic process as measured by the characteristics of their ARMA models to basic seimological parameters to provide a more realistic approach to hazard mapping.

2.EARTHQUAKE PROCESS MODELING

Stochastic process modeling of earthquake records has been investigated for some time. Traditional random process theory which deals with stationary processes has been used by defining a "period of strong motion" during which the process is considered stationary (4). The mean squared response for linear systems can be obtained and the fractiles of peak response can be estimated as a function of the averaging time. Although the real process is somewhat broader banded than the theoretical results assume, empirical corrections can be made (5).

A more realistic approach is simulation which was originally developed to complement measured records. A conceptually simple approach is to combine a white noise process with an appropriate filter to obtain a process with an acceptable power spectral density. Superposition of a series of sinusoids with amplitudes corresponding to the modified power spectrum and random phase angles yields a stationary process that can be analyzed over a short period of strong motion.

To simulate a truly nonstationary process, a filtered stationary process can be multiplied by a deterministic time dependent amplitude function. The result is a process with a more realistic time variation in rms acceleration (6,16). Alternatively, an evolutionary power spectrum can be used.

Most recently the application of auto-regressive moving average techniques (ARMA) has been investigated (17,18). The underlying stochastic process A(t) is developed in discrete steps from a recursive relationship of the form (3)

$$A(n) + c(1)*A(n-1) .. + c(N)*A(n-N) = U(n) + b(1)*U(n-1) .. + b(M)*U(n-M) \quad [1]$$

where U(n) is a series of independent zero mean random variables. The left side of Eq. 1 is known as the auto regressive part of order N while the right side is the moving average part of order M. The constants c(i), b(j) and the variance of the process U(n) are parameters of the process and depend on the order (N,M).

To estimate parameters, the total duration of a record is divided into moving segments of equal duration and the mean and variance in each segment are calculated. The amplitude at each point is normalized by subtracting the mean and dividing by the standard deviation for the segment centered on that point. The autocorrelation function of the resulting zero mean unit variance process is calculated and used to provide an initial estimate of the order (N,M). Initial estimates of the constants c(i) and b(j) are found by means of the Yule-Walker equations. Using these trial values in an iterative analysis, the final values of the constants and the variance of the white noise process U(t) are established so as to minimize the residuals of the process.

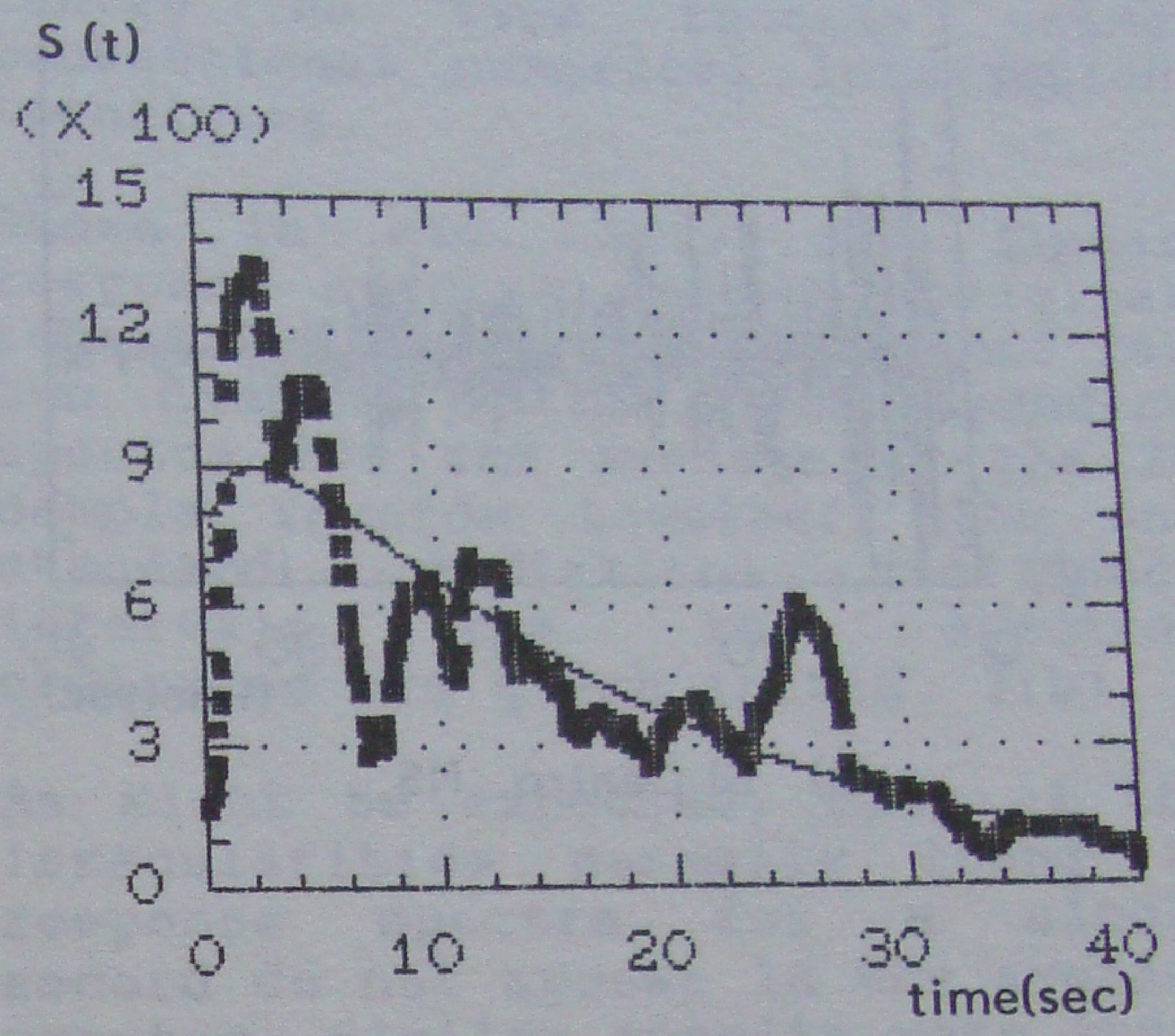
A critical element in analysis is to approximate the relationship between the standard deviation of the original process - the "envelope" function. To obtain a reasonable total number of parameters, a relatively simple expression must be assumed with constants again fitted by least squares.

Finally, to compare alternative values of the orders N and M as well as the assumed shape of the envelope function, the Akaike Information Criteria (8,9) is used.

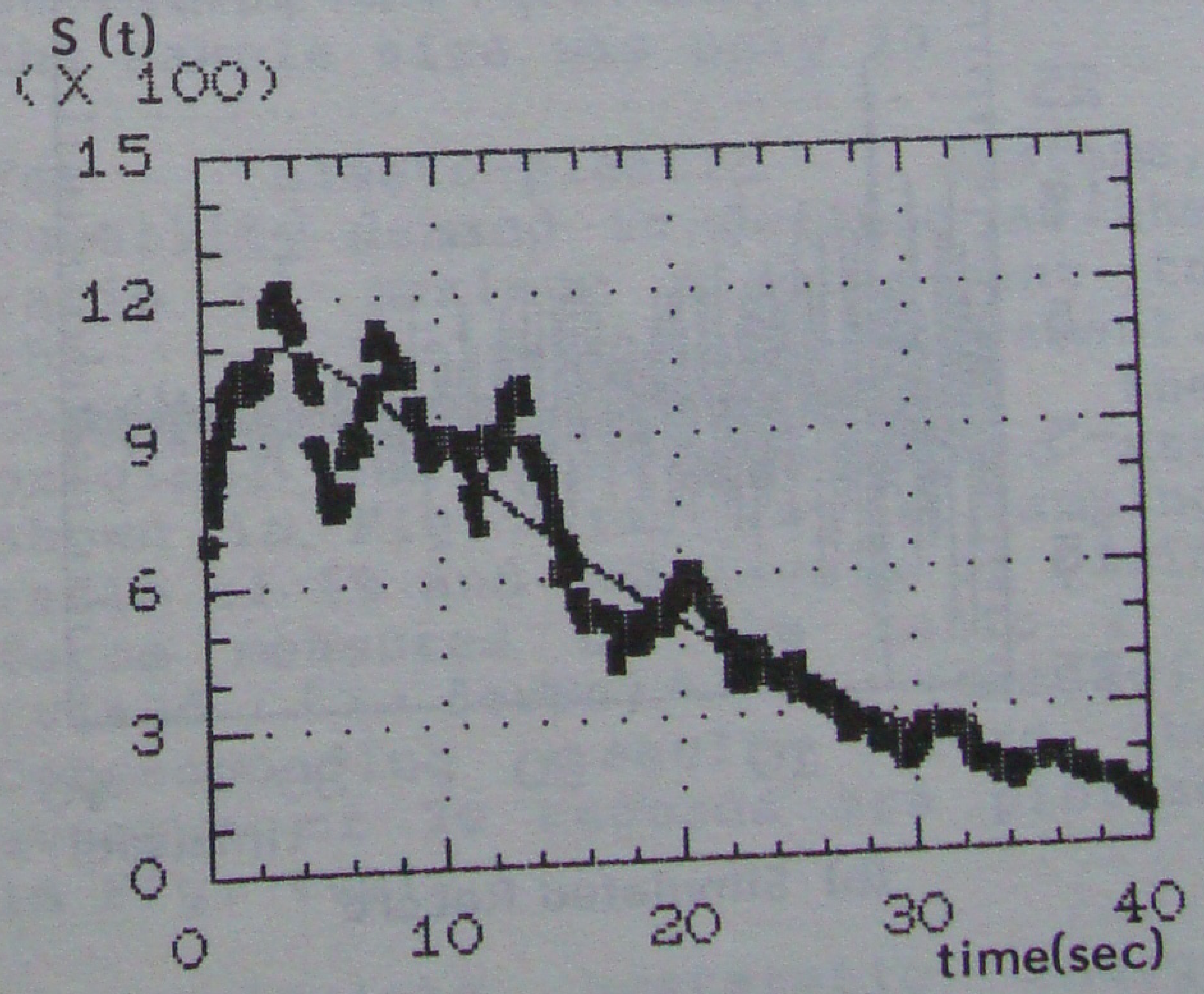
To obtain an artificial accelerogram, a white noise process Un with the desired variance is generated. A correlated zero mean unit variance process is then obtained recursively by means of Eq. 1. This process is then multiplied by the algebraic approximation to the original standard deviation or envelope function at each point in time to yield a nonstationary earthquake record.

Experience with these operations indicates that, as expected, the content of low frequency components below about 0.1/sec tend to be exaggerated. To correct this, a simple smoothing operation replacing the acceleration at each point by the average of three adjacent points and a base line correction to ensure near zero terminal velocity are applied (7).

As an application of the approach, El Centro NS was modeled using a basic time interval of 0.02 sec. to correspond with the published corrected data. A moving time segment of 20 sec. was used to



(a) El Centro NS

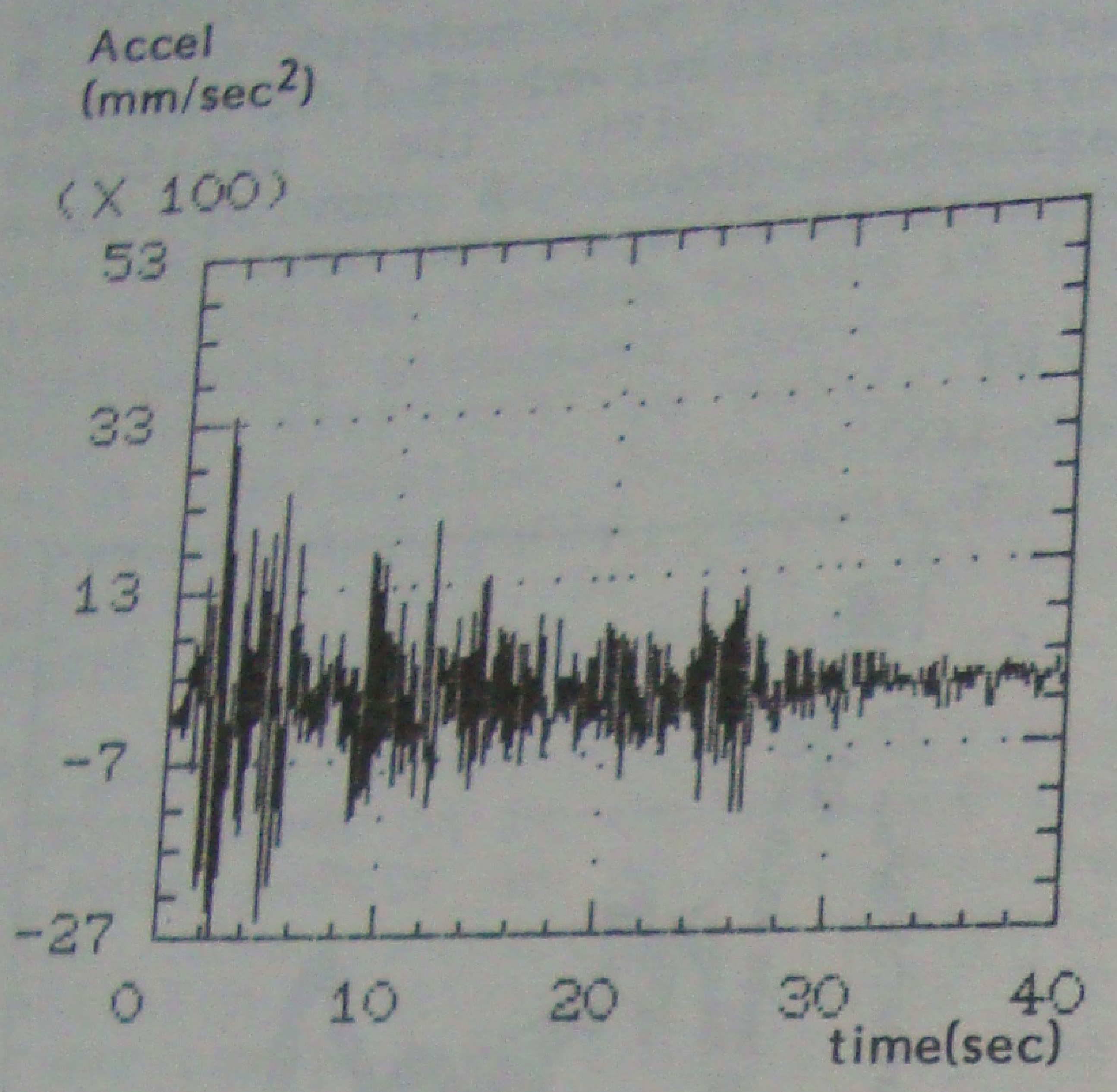


(b) Simulated Record

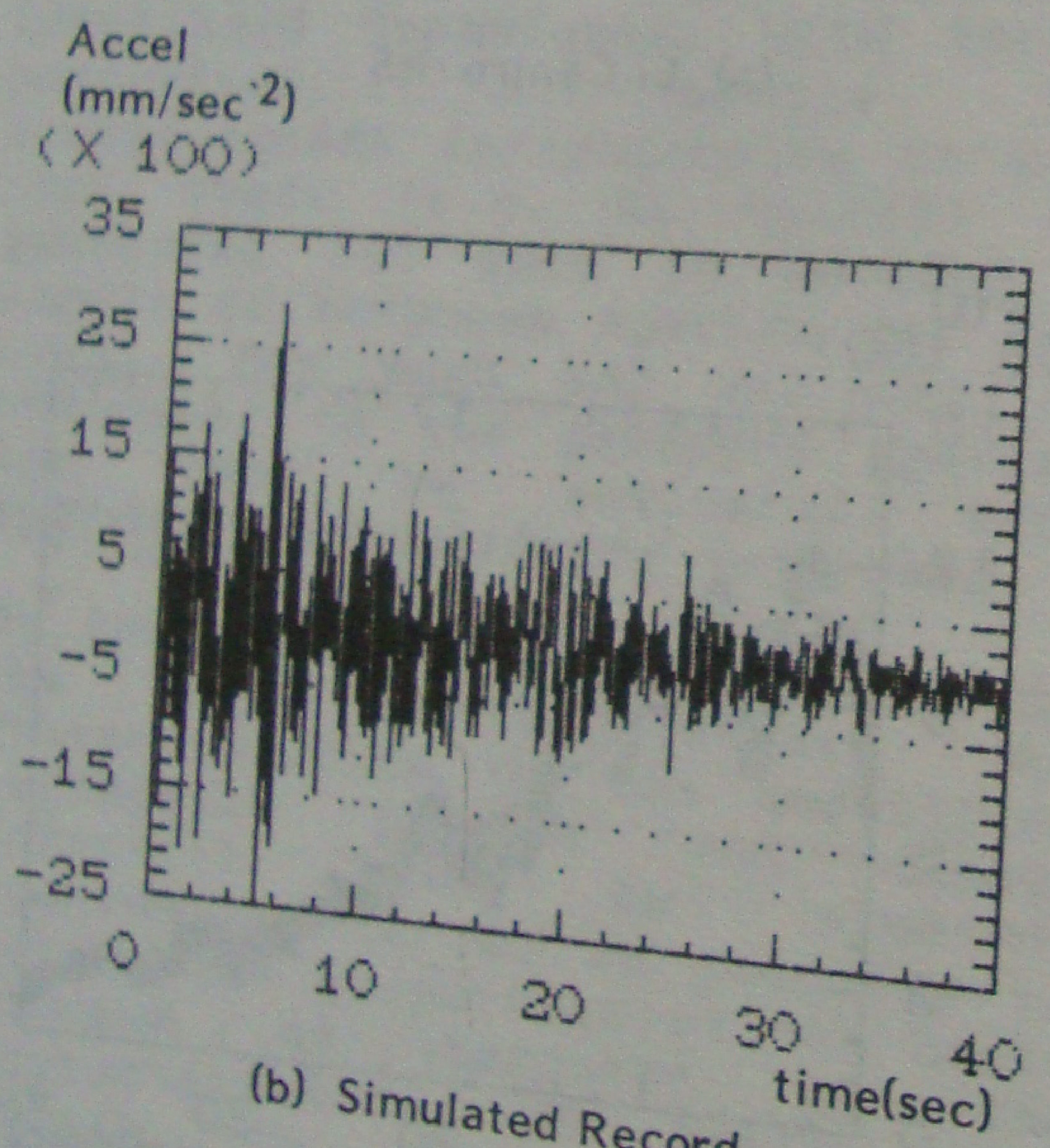
FIG. 1. ENVELOPE FUNCTIONS

calculate means which were very close to zero in all segments and the standard deviations or envelope function shown in Fig 1. The approximating function adopted was of the form

$$S(t) = A*tB*exp(-Ct) \qquad [2]$$



(a) El Centro NS



(b) Simulated Record

FIG.2 EARTHQUAKE RECORDS

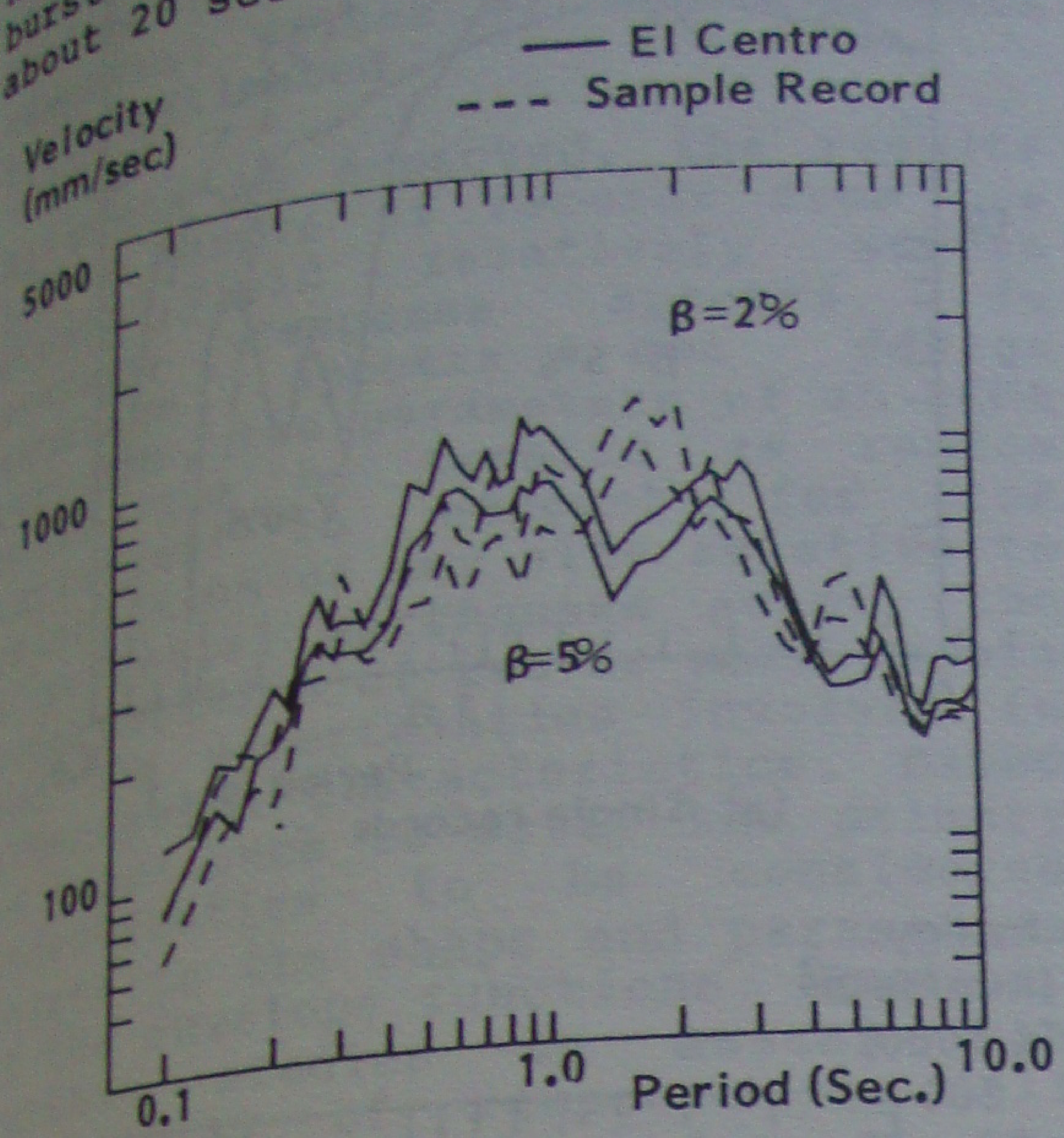
where the constants were found by least squares to be A = 927.7, B = 0.163 and C = 0.072. The approximating envelope function is also shown in Fig. 1. Application of the Yule-Walker equations, the least squares analysis and the Akaike Information Criteria yielded the process parameters N = 2, M = 1, c(1) = 1.243, c(2) = -0.455 and b(1) = 0.159. The variance of the white noise was 0.225.

Once the set of parameters has been established, generation of earthquake records is quite straightforward. The original record for El Centro NS and one artificial earthquake are shown in Figs 2. Shown in Table 1 are the rms accelerations, peak acceleration and peak absolute value of velocities for a sample of 20 El Centro NS records together with a comparison to the original.

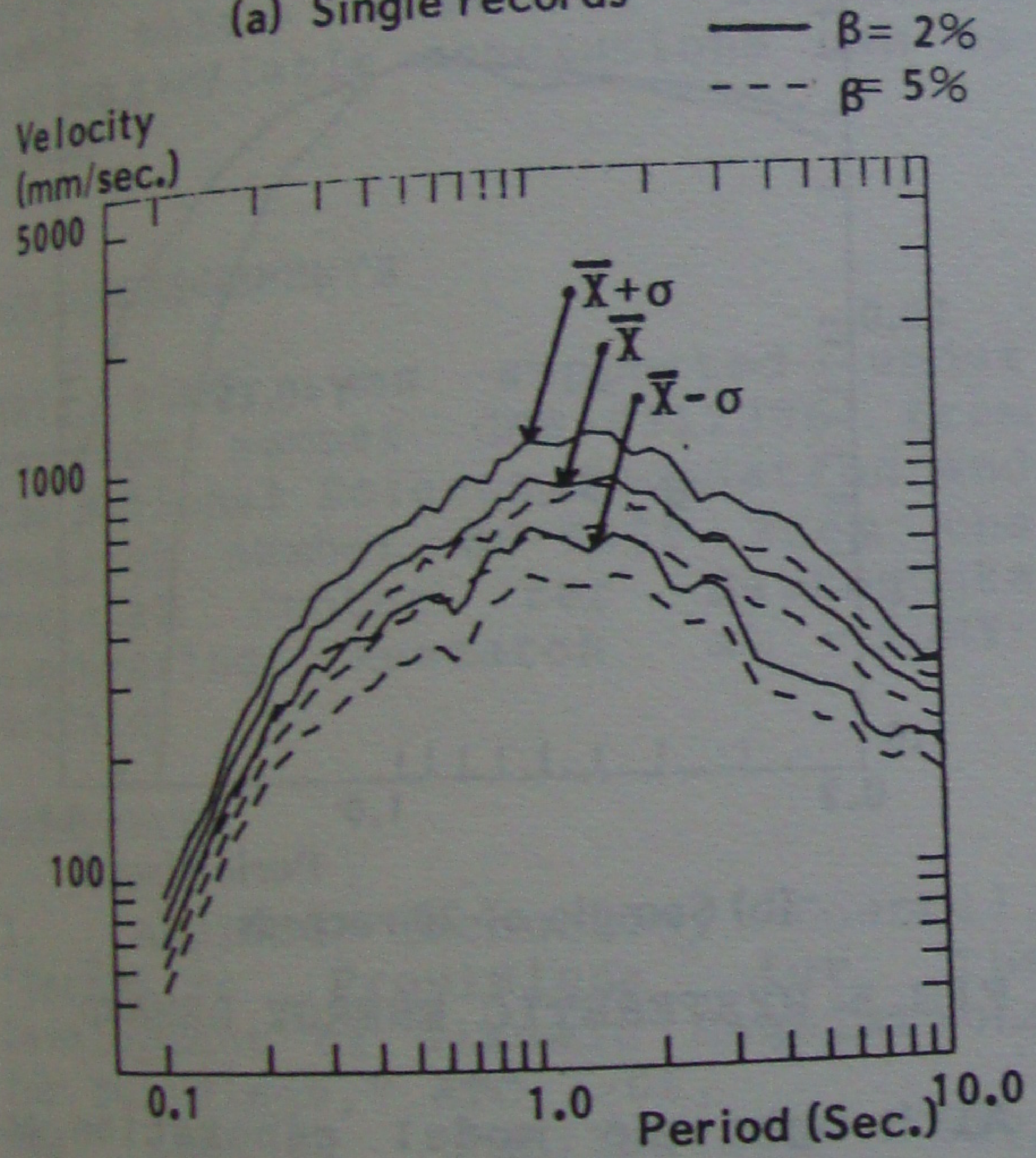
TABLE 1.
PROPERTIES OF GENERATED EARTHQUAKES

Record	rms A %g	peak A %g	peak V m-m/s
1	5.14	.331	560
2	4.83	.294	332
3	5.16	.238	562
4	5.00	.239	340
5	5.05	.259	408
6	5.16	.265	343
7	5.04	.226	349
8	5.00	.235	430
9	5.64	.328	565
10	5.22	.287	360
11	5.29	.261	318
12	5.28	.247	314
13	5.24	.267	407
14	4.82	.213	349
15	5.18	.244	331
16	4.97	.246	449
17	5.22	.262	361
18	5.00	.213	383
19	5.43	.260	463
20	4.88	.247	432
mean	5.12	.258	403
real	5.42	.348	381

For comparison purposes, an artificial record was analyzed as an original record to obtain the envelope function shown in Fig 1(b). In contrast to the measured record, the generated record does not show what seems to be a second burst of activity after a time of about 20 sec.



(a) Single records



(b) Sample of 20 records

FIG. 3 LINEAR RESPONSE SPECTRA

From the results obtained, it is clear that the artificial records have similar properties to the original except that peak values are somewhat smaller. Comparison of the envelope functions in Fig 1 suggests that the peak variances in the original record are not contained in the approximate expression of Eq. 2 which could account for the relatively low peaks. For practical purposes, the variability of the peak accelerations also seems to be rather low.

3. RESPONSE ANALYSIS

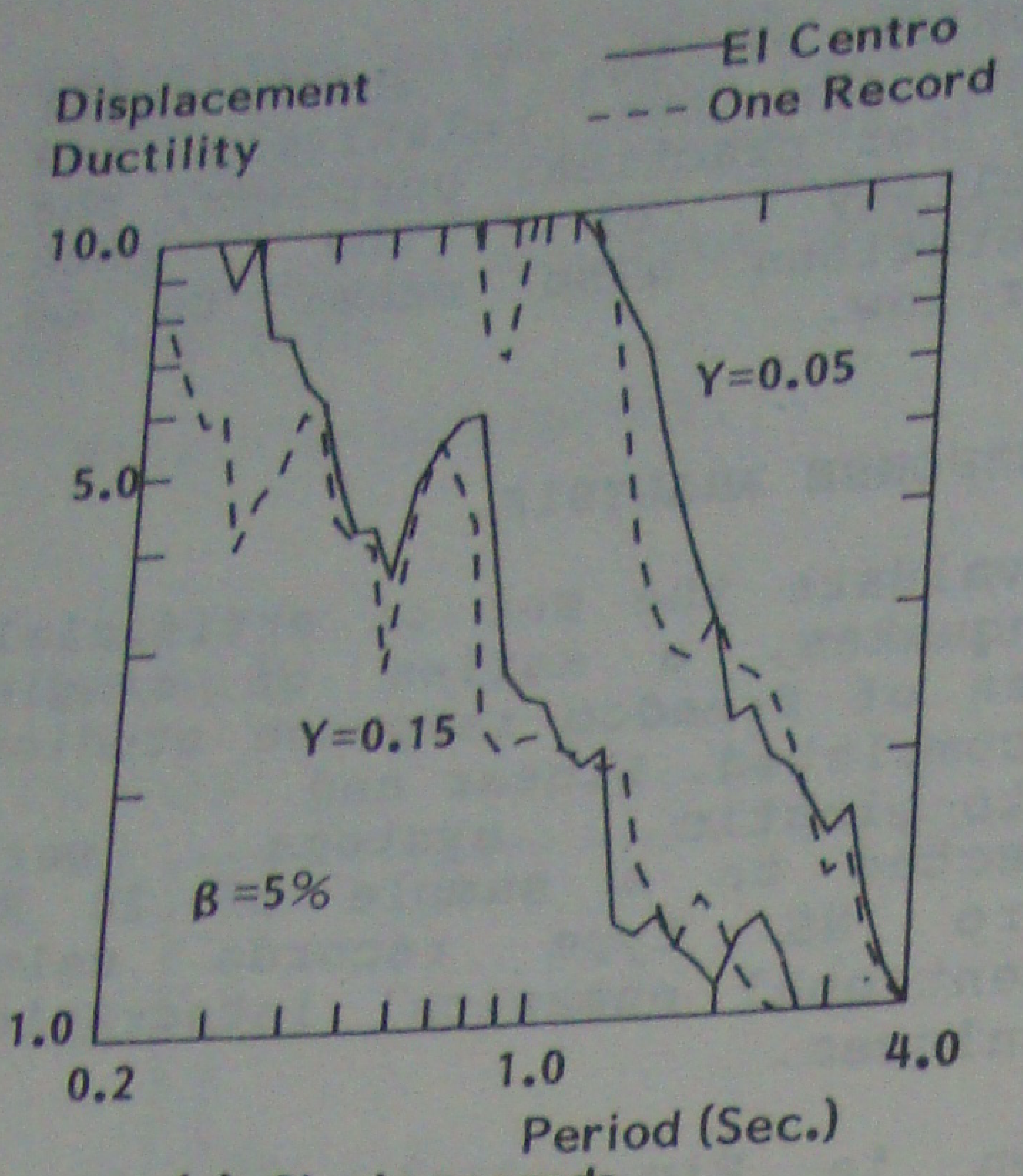
To evaluate the set of artificial earthquakes, a series of single degree of freedom response studies was completed. Linear and elasto-plastic systems were subjected to a sample of 20 El Centro NS type records using conventional numerical integration techniques.

Shown in Fig. 3(a) are linear response spectra for the original record and one generated record for two damping ratios. Corresponding spectra for the average of the 20 sample records together with one standard deviation confidence intervals for the spectral ordinates are given in Fig. 3(b).

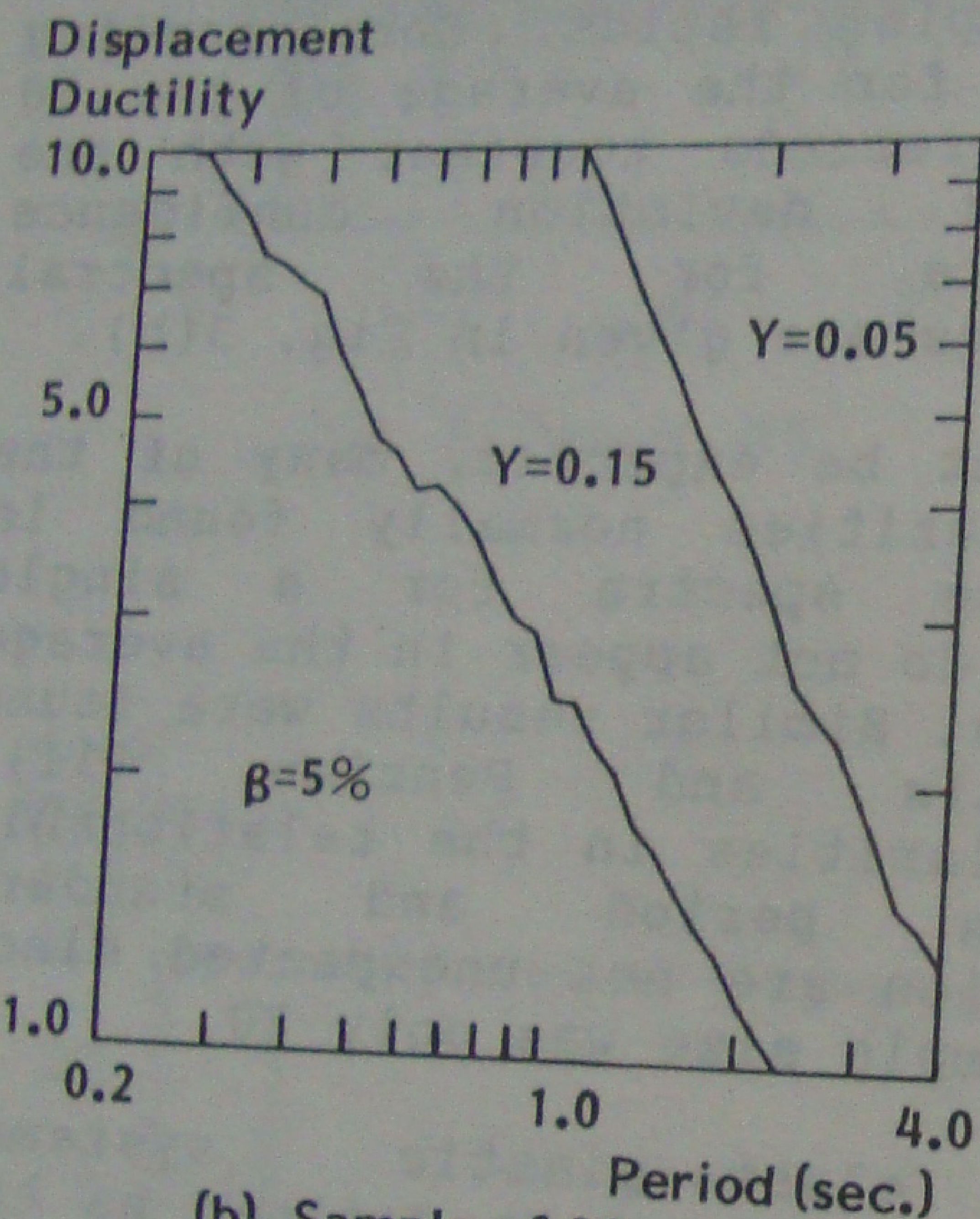
As might be expected, many of the irregularities normally found in response spectra for a single record do not appear in the average spectra. Similar results were found by Liu and Penzien (11). Irregularities in the relationship between period and standard deviation are not unexpected since the sample size was only 20.

For elasto-plastic systems, ductility demand is defined as the ratio of maximum displacement to the yield displacement. Corresponding spectra for the original and a sample record are shown in Fig. 4(a) for a damping ratio of 5% and two levels of yield force measured by the ratio Y = (yield force)/ (mass*g). Corresponding results for the average of 20 records are plotted in Fig. 4(b).

To calculate hysteretic energy demand, the cumulative area under the system response function is calculated (10). Spectra for two levels of yield force and a damping ratio of 5% are shown in Figs. 5. Once again, the avaerage spectra are much smoother than the spectra



(a) Single records



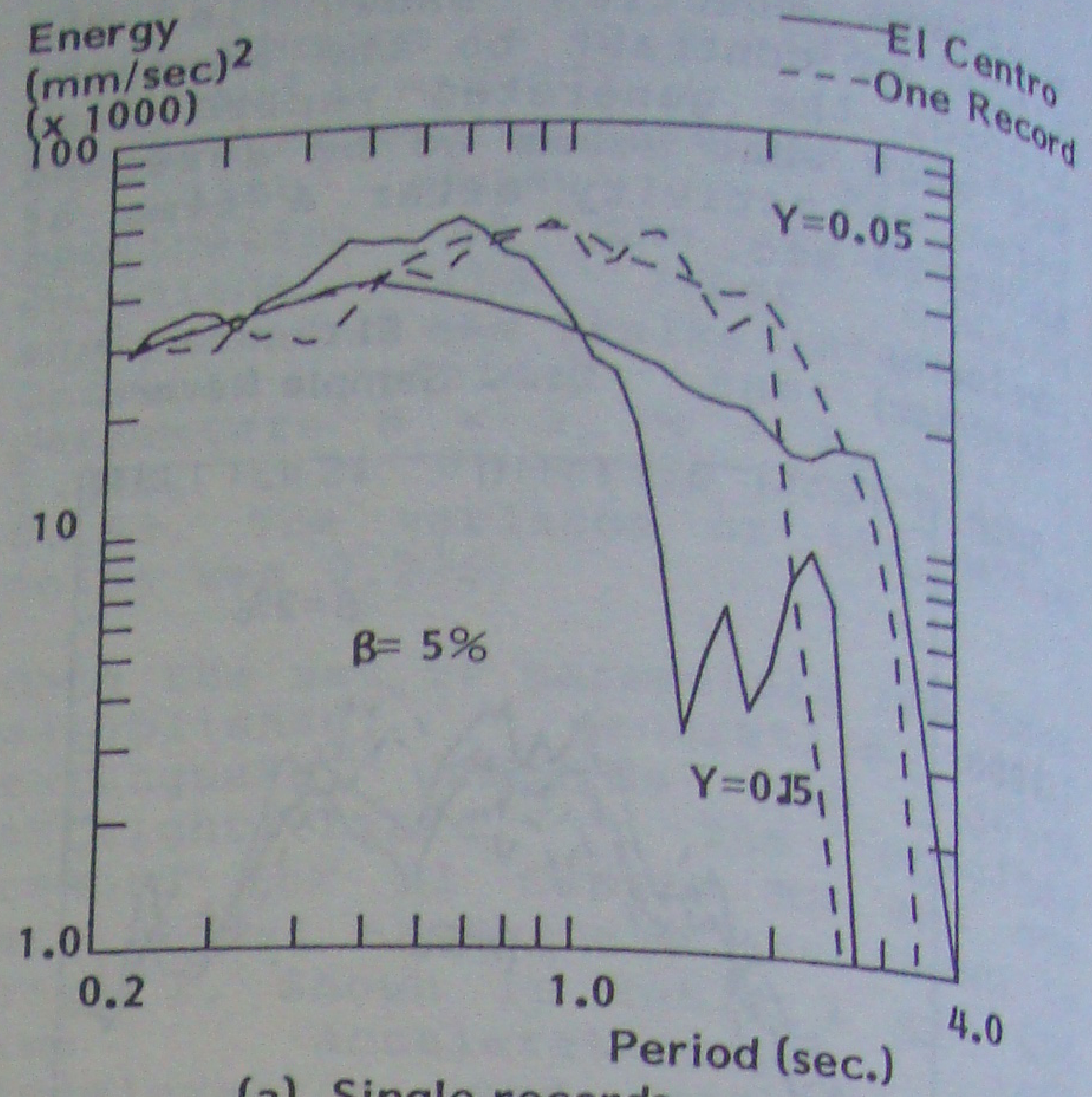
(b) Sample of 20 records

FIG.4 DISPLACEMENT DUCTILITY DEMAND

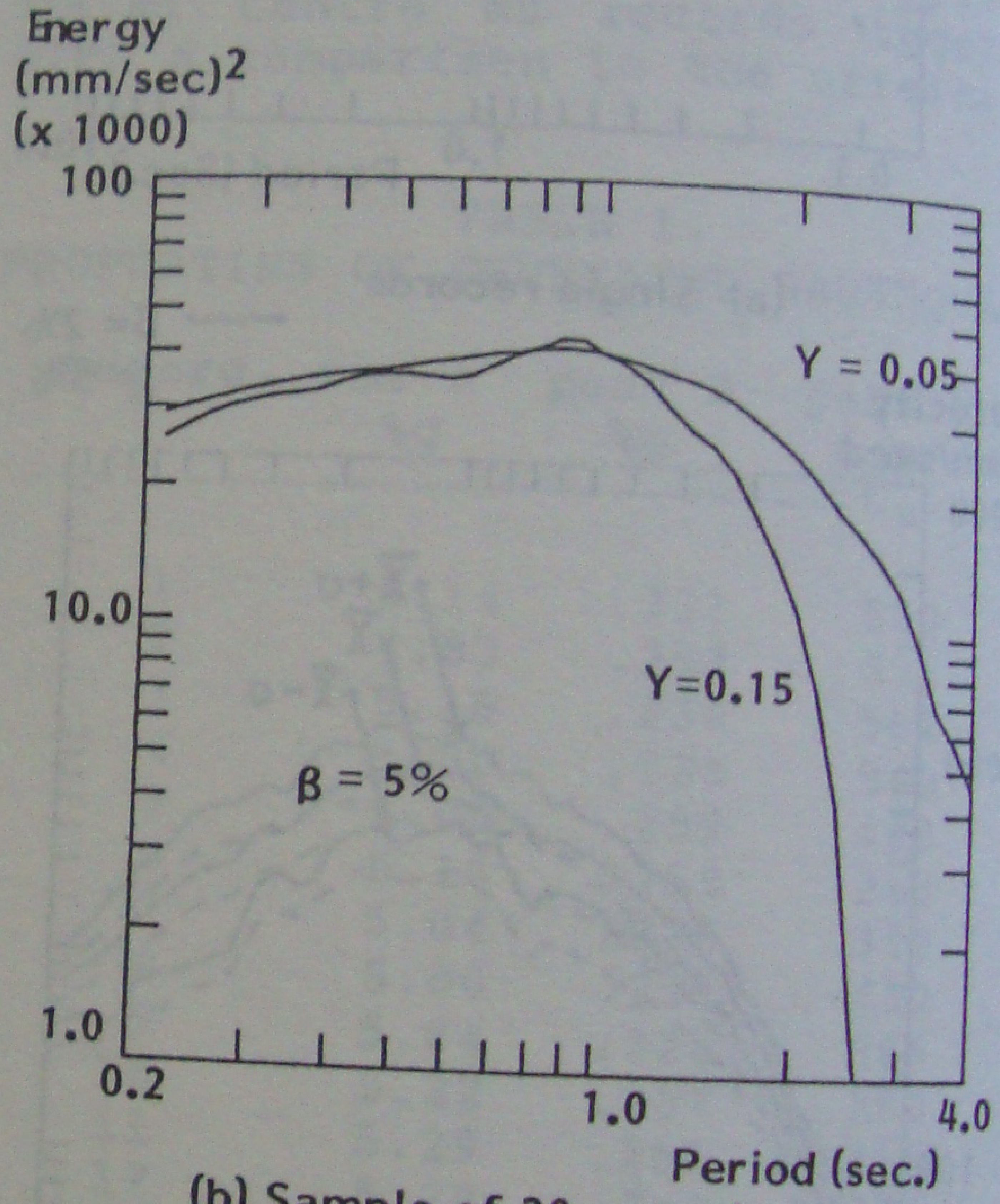
for individual records but have the same general pattern.

4. SUMMARY AND CONCLUSIONS

As an alternative to response spectra based on single records from several earthquakes, ARMA modeling techniques can be used to generate a sample of records for a single earthquake. This approach to simulation has the advantage that parameters can be estimated using maximum likelihood techniques. As a result, statistically valid samples can be generated for any seismic record.



(a) Single records



(b) Sample of 20 records

FIG.5 HYSTERETIC ENERGY DEMAND

Although the model generation and parameter estimation process involves a number of steps, all computations can be performed on a micro computer. Software packages for the analysis are available.

Experience to date suggests that the major uncertainties involved in applications are related to the shape of the envelope function. It seems that some earthquakes are characterized by more than one period of significant activity corresponding perhaps to the superposition of several closely spaced excitations. There are both theoretical and practical limitations on the number and numerical values of the constants to be estimated.

As might be expected, the results for a sample of records from one event yield relatively smooth average response spectra. To generate spectra for design purposes, the parameters of an ARMA model can be treated as random variables to account for the orientation of any site relative to a potential earthquake event, the superposition of two or more shocks and the uncertainties involved in attenuation characteristics. Based on experience to date, the primary uncertainties to be considered relate to the shape and parameters of the envelope functions. However, further studies to assess the sensitivity of response to all parameters and the order of the ARMA models must be completed before reliable conclusions can be drawn.

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